Chapter 7
Currency Options & Options Markets

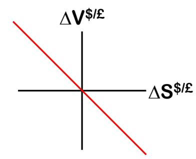
A forward obligation

- A £1 million obligation due in four months

Underlying transaction

-£1,000,000

Currency exposure



A forward hedge

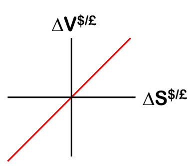
- Buy £1 million in the forward market at the forward price $F_1^{\$/£}$ = $1.45/£

Exposure of forward contract

Long pound forward

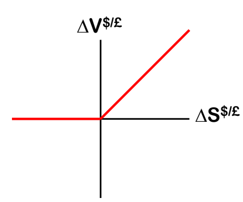
+£1,000,000

-$1,450,000



An option hedge

- A currency option is like one-half of a forward contract
  - the option holder gains if pound sterling rises
  - the option holder does not lose if pound sterling falls



Long pound call
(option to buy pound sterling)

CME pound Dec 1450 call (American)

- **Type of option**: a **call option** to **buy** pounds
- **Underlying asset**: CME December pound sterling futures contract
- **Contract size**: £62,500
- **Expiration date**: 3rd week of December
- **Exercise price**: $1.45/£
- **Rule for exercise**: an **American** option exercisable anytime until expiration

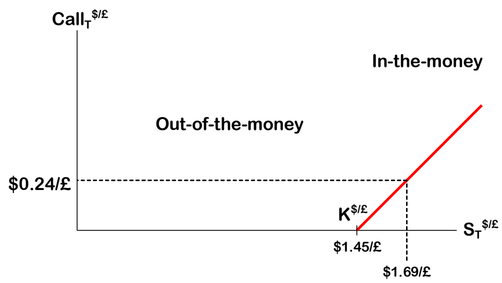
Currency option quotations

British pound (CME)
£62,500; cents per pound

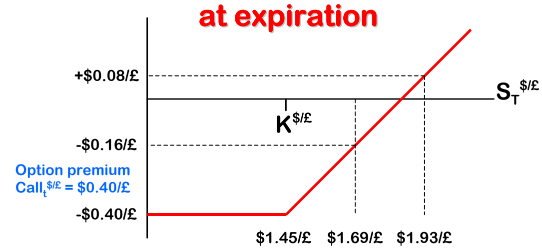
Strike	Calls-Settle			Puts-Settle		
Price	Oct	Nov	Dec	Oct	Nov	Dec
1430	2.38	. . . .	2.78	0.39	0.61	0.80
1440	1.68	1.94	2.15	0.68	0.94	1.16
1450	1.12	1.39	1.61	1.12	1.39	1.61
1460	0.69	0.95	1.17	1.69	1.94	2.16
1470	0.40	0.62	0.82	2.39	. . . .	2.80

Note: $S_0^{\$/£}$ = $1.45/£

Payoff profile of a pound call at expiration



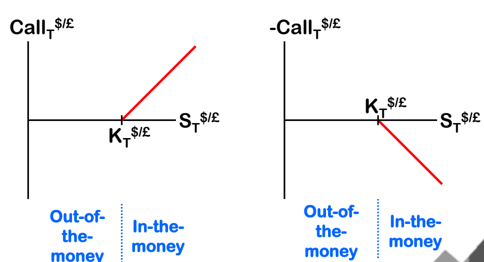
Profit (loss) on a call option at expiration



FX rate at expiration	$1.45/£	$1.69/£	$1.93/£
Premium (cost)	-$25,000	-$25,000	-$25,000
Exercise price	$0	-$90,625	-$90,625
Spot £ sale	$0	$105,625	$120,625
Net profit	-$25,000	-$10,000	+$5,000

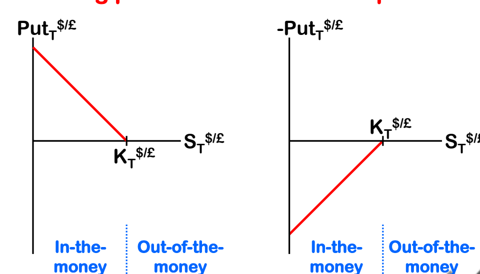
Payoff profile of a call option at expiration

Long call | **Short call**



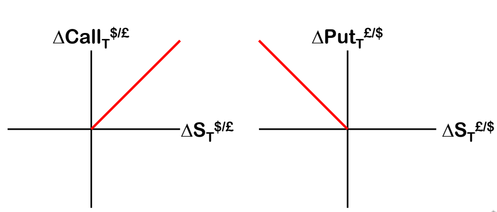
Payoff profile of a put option at expiration

Long put | **Short put**



Puts and calls

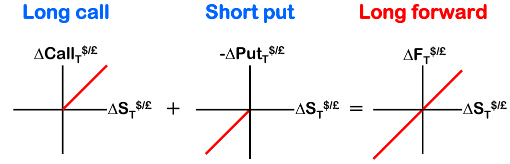
An option to buy pounds at $K_T^{\$/£}$ | **An option to sell dollars at $K_T^{£/\$}$**



Forwards, puts, and calls

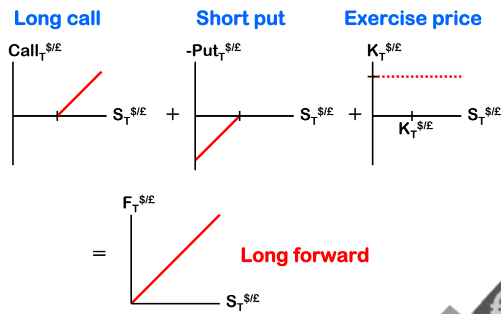
- A combination of a **long call** and a **short put** at the same exercise price and with the same expiration date results in a **long forward** position at that forward price

Long call | **Short put** | **Long forward**



Put-call parity:

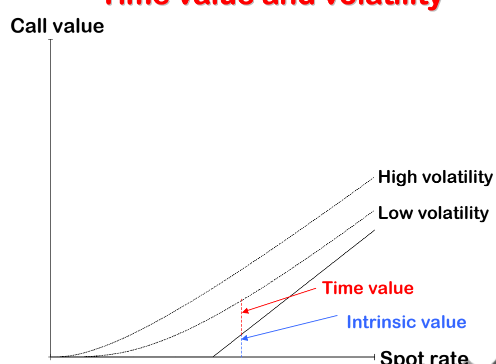
$Call_T^{d/f} - Put_T^{d/f} + K^{d/f} = F_T^{d/f}$



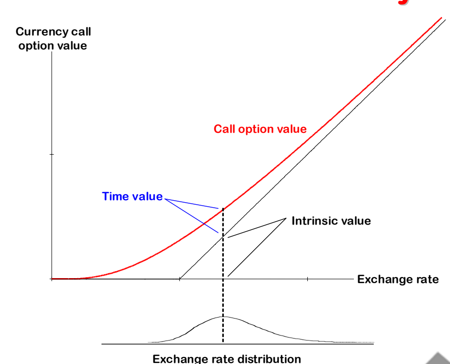
The time value of an option

- **Time value** = Option value - intrinsic value
  - **Intrinsic value** = value if exercised immediately
- The **time value** of a currency option is a function of the following six determinants
  - **Exchange rate** underlying the option
  - **Exercise price** or striking price
  - **Riskless rate** of interest i^d in currency d
  - **Riskless rate** of interest i^f in currency f
  - **Volatility** in the underlying exchange rate
  - **Time to expiration**

Time value and volatility



Time value and volatility



The interaction of time and variance

- If instantaneous changes are a random walk, then T-period variance is T times one-period variance

$$\sigma_T^2 = T\,\sigma^2$$

where σ^2 = 1-period variance
σ_T^2 = T-period variance

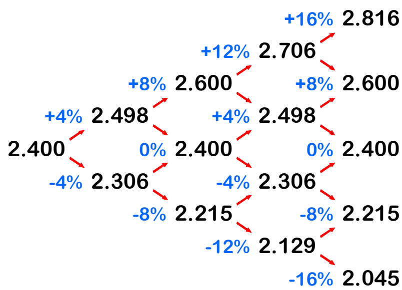
- Estimation of exchange rate volatility
  - Historical volatility
  - Implied volatility

Advanced: Pricing currency options

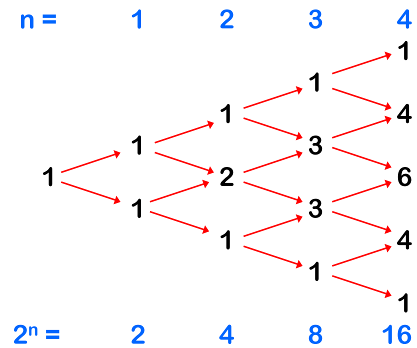
- Suppose the Australian-per-US dollar spot rate A$2.4/$ **bifurcates** by a continuously compounded ±4 percent per period for 4 periods
- 4 successive bifurcations result in 2^4 = 16 price paths
- Value after 1 period is $P_1 = P_0 e^{\pm 0.04}$
  - (A$2.4/$)$e^{-0.04}$ = A$2.306/$
  - (A$2.4/$)$e^{+0.04}$ = A$2.498/$

  each with 50 percent probability

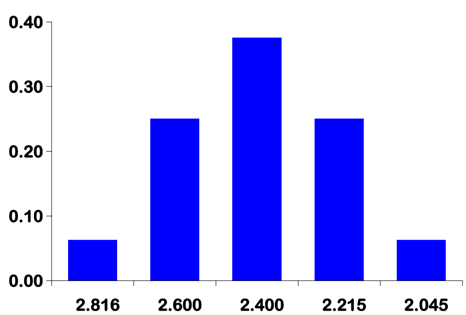
±4 percent for 4 periods



2^4 = 16 possible price paths



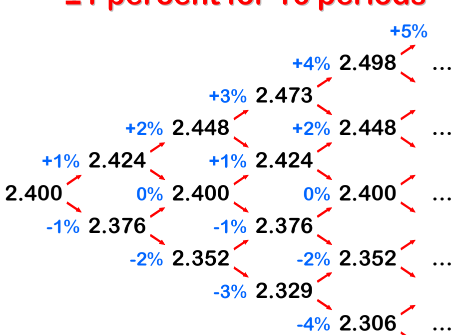
End-of-period distribution for n = 4



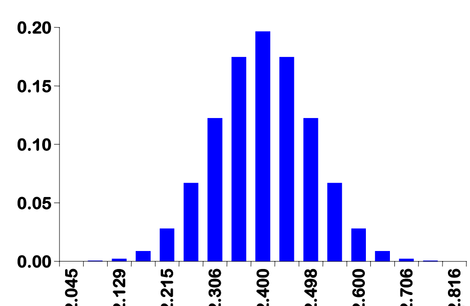
More frequent compounding…

- Suppose we apply the binomial model with 1% per period for 16 periods
- This results in 2^{16} = 65,536 price paths and (n+1) = 17 possible prices

±1 percent for 16 periods



End-of-period distribution for n = 16



The Binomial and B-S OPMs

- As the binomial process generating up and down movements bifurcates over shorter and shorter intervals
  - the binomial distribution approaches the normal distribution
  - continuous-time pricing methods (e.g., the Black-Scholes OPM) can be used