Chapter 5
The International Parity Conditions

Prices

Prices appear as upper case symbols

- P_t^d = price of an asset at time t in currency d
- $S_t^{d/f}$ = spot exchange rate at time t in currency d
- $F_t^{d/f}$ = forward exchange rate between currencies d and f
- E[...] = expectation operator (e.g. $E[S_t^{€/\$}]$)

Rates of change

Changes in a price appear as lower case symbols

- r_t^d = an asset's return in currency d during period t
- p_t^d = inflation in currency d in period t
- ι_t^d = real interest rate in currency d in period t
- $s_t^{d/f}$ = change in the spot rate during period t

The law of one price

Equivalent assets sell for the same price

(also called **purchasing power parity**, or **PPP**)

- Seldom holds for nontraded assets
- Can't compare assets that vary in quality
- May not hold precisely when there are market frictions

An example: The world price of gold

Suppose $P^£$ = £250/oz in London
$P^€$ = €400/oz in Berlin

The law of one price requires:

$$P_t^£ = P_t^€ \, S_t^{£/€}$$

⇒ £250/oz = (€400/oz) (£0.6250/€)
or 1/(£0.6250/€) = €1.6000/£

- If this relation does not hold, then there is an opportunity to lock in a riskless arbitrage profit.

An example with transactions costs

Gold dealer A

£250.25/oz Offer
£250.00/oz Bid

Buy low from A

FX dealer

€1.599/£ bid
€1.601/£ ask

Gold dealer B

€401.40/oz Offer
€401.00/oz Bid

Sell high to B

Arbitrage profit

Pay £250.25 million to buy 1 million oz from A

+1 million oz
-£250,250,000

Sell 1 million oz to B for €401 million

+€401,000,000
-1 million oz

Buy £s with €s at the spot rate

+£250,468,500
-€401,000,000

Arbitrage profit €218,500

Cross exchange rate equilibrium

$$S^{d/e} S^{e/f} S^{f/d} = 1$$

If $S^{d/e}S^{e/f}S^{f/d} < 1$, then either $S^{d/e}$, $S^{e/f}$ or $S^{f/d}$ must rise

⇒ For each spot rate, buy the currency in the denominator with the currency in the numerator

If $S^{d/e}S^{e/f}S^{f/d} > 1$, then either $S^{d/e}$, $S^{e/f}$ or $S^{f/d}$ must fall

⇒ For each spot rate, sell the currency in the denominator for the currency in the numerator

Kirt C. Butler, *Multinational Finance*, South-Western College Publishing, 3e

A cross exchange rate table

	£	C$	€	¥	SFr	$
UK pound	1.000	0.402	0.659	0.0052	0.451	0.622
Canadian $	2.487	1.000	1.634	0.0130	1.120	1.546
Euro	1.518	0.612	1.000	0.0079	0.685	0.947
Japanese yen	191.6	77.24	126.1	1.0000	86.48	119.4
Swiss Franc	2.221	0.893	1.460	0.0116	1.000	1.381
US Dollar	1.609	0.647	1.057	0.0084	0.724	1.000

Kirt C. Butler, *Multinational Finance*, South-Western College Publishing, 3e

Cross exchange rates and triangular arbitrage

Suppose

$S^{Rbl/\$}$ = Rbl 5.000/\$ ⇔ $S^{\$/Rbl}$ = \$0.2000/Rbl

$S^{\$/¥}$ = \$0.01000/¥ ⇔ $S^{¥/\$}$ = ¥100.0/\$

$S^{¥/Rbl}$ = ¥20.20/Rbl ⇔ $S^{Rbl/¥}$ ≈ Rbl 0.04950/¥

$S^{Rbl/\$}\ S^{\$/¥}\ S^{¥/Rbl}$

= (Rbl 5/\$)(\$.01/¥)(¥20.20/Rbl)

= 1.01 > 1

Kirt C. Butler, *Multinational Finance*, South-Western College Publishing, 3e

Cross exchange rates and triangular arbitrage

$S^{Rbl/\$}\ S^{\$/¥}\ S^{¥/Rbl}$ = 1.01 > 1

⇒ Currencies in the denominators are too high relative to the numerators, so

sell dollars and buy rubles
sell yen and buy dollars
sell rubles and buy yen

Kirt C. Butler, *Multinational Finance*, South-Western College Publishing, 3e

An example of triangular arbitrage

$S^{Rbl/\$}\ S^{\$/¥}\ S^{¥/Rbl}$ = 1.01 > 1

Sell \$1 million and buy Rbl 5 million
Sell ¥100 million yen and buy \$1 million
Sell Rbl 4.950 million and buy ¥100 million

⇒ Profit of 50,000 rubles
= \$10,000 at Rbls5.000/\$
or 1% of the initial amount

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International parity conditions that span both currencies and time

Interest rate parity	Less reliable linkages
$F_t^{d/f} / S_0^{d/f} = [(1+i^d)/(1+i^f)]^t$	$= E[S_t^{d/f}] / S_0^{d/f}$
	$= [(1+p^d)/(1+p^f)]^t$

where

$S_0^{d/f}$ = today's spot exchange rate

$E[S_t^{d/f}]$ = expected future spot rate

$F_t^{d/f}$ = forward rate for time t exchange

i = a country's nominal interest rate

p = a country's inflation rate

Interest rate parity

$$F_t^{d/f}/S_0^{d/f} = [(1+i^d)/(1+i^f)]^t$$

- Forward premiums and discounts are entirely determined by interest rate differentials.
- This is a parity condition that you can trust.

Interest rate parity: Which way do you go?

If $F_t^{d/f}/S_0^{d/f} > [(1+i^d)/(1+i^f)]^t$

then	so...
$F_t^{d/f}$ must fall	Sell f at $F_t^{d/f}$
$S_0^{d/f}$ must rise	Buy f at $S_0^{d/f}$
i^d must rise	Borrow at i^d
i^f must fall	Lend at i^f

Interest rate parity: Which way do you go?

If $F_t^{d/f}/S_0^{d/f} < [(1+i^d)/(1+i^f)]^t$

then	so...
$F_t^{d/f}$ must rise	Buy f at $F_t^{d/f}$
$S_0^{d/f}$ must fall	Sell f at $S_0^{d/f}$
i^d must fall	Lend at i^d
i^f must rise	Borrow at i^f

Interest rate parity is enforced through "covered interest arbitrage"

An Example:

Given: $i^{\$} = 7\%$ $S_0^{\$/£} = \$1.20/£$

$i^{£} = 3\%$ $F_1^{\$/£} = \$1.25/£$

$F_1^{\$/£} / S_0^{\$/£} > (1+i^{\$}) / (1+i^{£})$

$1.041667 > 1.038835$

The fx and Eurocurrency markets are not in equilibrium.

Covered interest arbitrage

1. Borrow \$1,000,000 at $i^{\$} = 7\%$ — +\$1,000,000; -\$1,070,000
2. Convert \$s to £s at $S_0^{\$/£} = \$1.20/£$ — +£833,333; -\$1,000,000
3. Invest £s at $i^{£} = 3\%$ — +£858,333; -£833,333
4. Convert £s to \$s at $F_1^{\$/£} = \$1.25/£$ — +\$1,072,920; -£858,333
5. Take your profit:
   ⇒ \$1,072,920–\$1,070,000 = \$2,920

Forward rates as predictors of future spot rates

$F_t^{d/f} = E[S_t^{d/f}]$

or

$F_t^{d/f} / S_0^{d/f} = E[S_t^{d/f}] / S_0^{d/f}$

Forward rates are unbiased estimates of future spot rates.

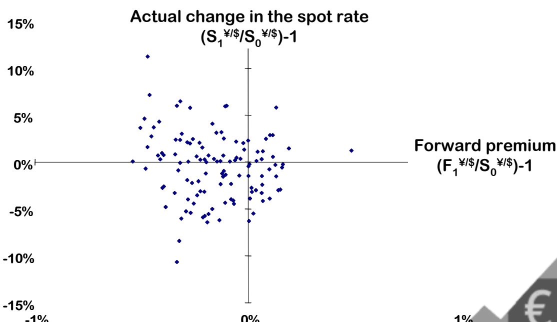
Forward rates as predictors of future spot rates

$E[S_t^{d/f}] / S_0^{d/f} = F_t^{d/f} / S_0^{d/f}$

Speculators will force this relation to hold on average

- For daily exchange rate changes, the best estimate of tomorrow's spot rate is the current spot rate
- As the sampling interval is lengthened, the performance of forward rates as predictors of future spot rates improves

The one-month ¥/$ forward rate as a predictor of the future spot rate



Relative purchasing power parity (RPPP)

Let P_t = a consumer price index level at time t

Then inflation $p_t = (P_t - P_{t-1}) / P_{t-1}$

$$E[S_t^{d/f}] / S_0^{d/f} = (E[P_t^d] / E[P_t^f]) / (P_0^d / P_0^f)$$
$$= (E[P_t^d]/P_0^d) / (E[P_t^f]/P_0^f)$$
$$= (1+E[p^d])^t / (1+E[p^f])^t$$

where p^d and p^f are geometric mean inflation rates.

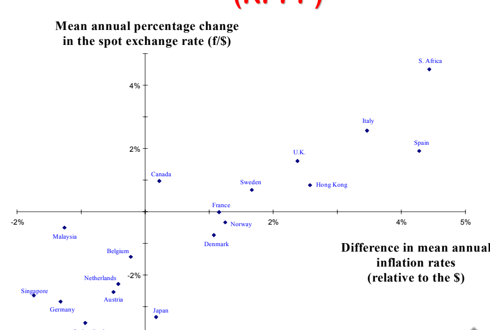
Relative purchasing power parity (RPPP)

$E[S_t^{d/f}] / S_0^{d/f} = (1+E[p^d])^t / (1+E[p^f])^t$

Speculators will force this relation to hold on average

- The expected change in a spot exchange rate should reflect the difference in inflation between the two currencies.
- This relation only holds over the long run.

Relative purchasing power parity (RPPP)



International Fisher relation
(Fisher Open hypothesis)

$[(1+i^d)/(1+i^f)]^t = [(1+p^d)/(1+p^f)]^t$

Recall the Fisher relation: $(1+i) = (1+\iota)(1+p)$

If real rates of interest are equal across currencies, then

$[(1+i^d)/(1+i^f)]^t = [(1+\iota^d)(1+p^d)]^t / [(1+\iota^f)(1+p^f)]^t$

$= [(1+p^d)/(1+p^f)]^t$

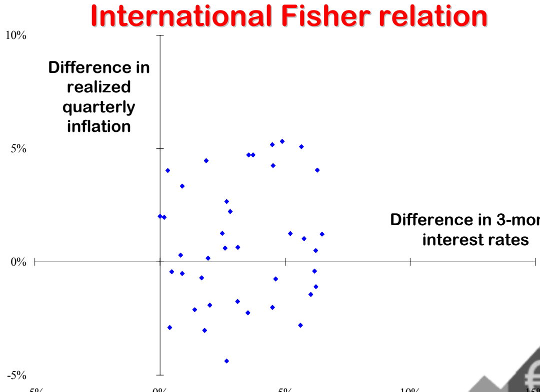
International Fisher relation
(Fisher Open hypothesis)

$[(1+i^d)/(1+i^f)]^t = [(1+p^d)/(1+p^f)]^t$

Speculators will force this relation to hold on average

- **If real rates of interest are equal across countries ($\iota^d = \iota^f$), then interest rate differentials merely reflect inflation differentials**
- **This relation is unlikely to hold at any point in time, but should hold in the long run**

International Fisher relation



Summary: Int'l parity conditions

International Fisher relation

Interest rates $[(1+i^d)/(1+i^f)]^t$		Inflation rates $[(1+p^d)/(1+p^f)]^t$
Interest rate parity		**Relative PPP**
$F_t^{d/f} / S_0^{d/f}$ Forward-spot differential		$E[S_t^{d/f}] / S_0^{d/f}$ Expected change in the spot rate

Forward rates as predictors of future spot rates

Purchasing power (dis)parity
The Big Mac Index

	P^f	$S^{f/\$}$	$P^\$$	Relative to U$ price
USA ($)	2.50	1.000	2.50	1.00
Britain (£)	2.00	0.6250	3.20	1.28
Euro-zone (€)	2.60	1.100	2.36	0.95
Japan (¥)	300	125.0	2.40	0.96
S. Korea (Won)	3000	1250	2.40	0.96
Switzerland (SFr)	6.5	1.500	4.33	1.73
Taiwan (NT$)	70.0	35.00	2.00	0.80

The real exchange rate

- **The real exchange rate adjusts the nominal exchange rate for differential inflation since an arbitrarily defined base period**

Change in the nominal exchange rate

Example

$S_0^{¥/\$}$ = ¥100/$

$S_1^{¥/\$}$ = ¥110/$

$E[p^{¥}]$ = 0%

$E[p^{\$}]$ = 10%

$s_1^{¥/\$} = (S_1^{¥/\$} – S_0^{¥/\$})/S_0^{¥/\$} = 0.10$,
or a 10 percent nominal change

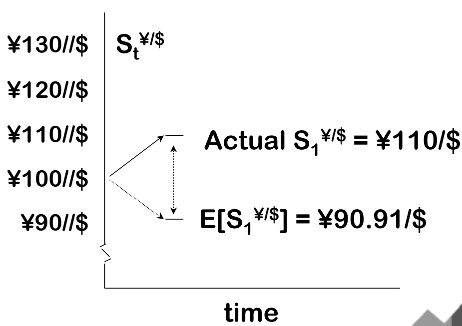
The expected nominal exchange rate

But RPPP implies

$E[S_1^{¥/\$}] = S_0^{¥/\$} (1+ p^{¥})/(1+ p^{\$})$
= ¥90.91/$

What is the change in the nominal exchange rate relative to the expectation of ¥90.91/$?

Actual versus expected change



Change in the real exchange rate

- In real (or purchasing power) terms, the dollar has appreciated by

  (¥110/$) / (¥90.91/$) – 1 = +0.21

  or 21 percent more than expected

Change in the real exchange rate

$$(1+x_t^{d/f}) = (S_t^{d/f} / S_{t-1}^{d/f}) [(1+p_t^{f})/(1+p_t^{d})]$$

where

$x_t^{d/f}$ = percentage change in the real exchange rate

$S_t^{d/f}$ = the nominal spot rate at time t

p_t^{c} = inflation in currency c during period t

Change in the real exchange rate

Example $S_0^{¥/\$}$ = ¥100/$ ➜ $S_1^{¥/\$}$ = ¥110/$
$E[p^{¥}]$ = 0% and $E[p^{\$}]$ = 10%

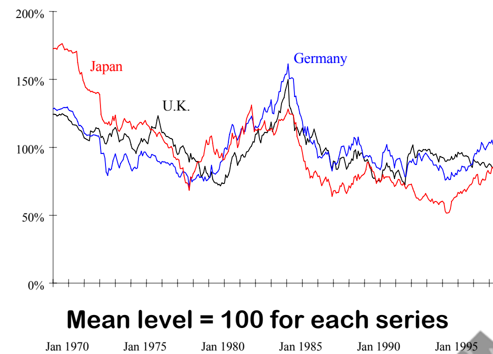
$x_t^{¥/\$}$ = [(¥110/$)/(¥100/$)][1.10/1.00] -1
= 0.21,

or a 21 percent increase in real purchasing power

Behavior of real exchange rates

- Deviations from purchasing power parity
  - can be substantial in the short run
  - and can last for several years
- Both the level and variance of the real exchange rate are autoregressive

Real value of the dollar (1970-1998)



Appendix 5-A
Continuous time finance

- Most theoretical and empirical research in finance is conducted in continuously compounded returns

Holding period returns are asymmetric

$r_1 = +100\%$ 200 $r_2 = -50\%$

100 → 200 → 100

$(1+r_{TOTAL}) = (1+r_1)(1+r_2)$

$= (1+1)(1-½) = (2)(½) = 1$

$\Rightarrow r_{TOTAL} = 0\%$

Continuous compounding

Let

r = holding period (e.g. annual) return

r = *continuously compounded return*

$r = \ln(1+r) = \ln(e^{r}) \Leftrightarrow (1 + r) = e^{r}$

where

ln(.) is the natural logarithm

with base $e \approx 2.718$

Continuous returns are symmetric

+69.3%   200   -69.3%

100 → 200 → 100

$r_{TOTAL} = \ln[(1+r_1)(1+r_2)]$

$= r_1 + r_2 = +0.693 - 0.693 = 0.000$

$\Rightarrow r_{TOTAL} = 0\%$

Properties of natural logarithms
(for x > 0)

$e^{\ln(x)} = \ln(e^x) = x$

$\ln(AB) = \ln(A) + \ln(B)$

$\ln(A^t) = t * \ln(A)$

$\ln(A/B) = \ln(AB^{-1})$

$= \ln(A) - \ln(B)$

Continuously compounded returns are additive rather than multiplicative

$\ln[(1+r_1)(1+r_2) \dots (1+r_T)]$

$= r_1 + r_2 + \dots + r_T$

The international parity conditions in continuous time

Over a single period

$\ln(F_1^{d/f} / S_0^{d/f}) = i^d - i^f$

$= E[p^d] - E[p^f]$

$= E[s^{d/f}]$

where $s^{d/f}$, p^d, p^f, i^d, and i^f are continuously compounded

The international parity conditions in continuous time

Over t periods

$\ln(F_t^{d/f} / S_0^{d/f}) = t(i^d - i^f)$

$= t(E[p^d] - E[p^f])$

$= t\,E[s^{d/f}]$

where $s^{d/f}$, p^d, p^f, i^d, and i^f are continuously compounded