



APPENDIX

13A

DEGREE OF LEVERAGE

In our discussion of operating leverage in Chapter 13, we made no mention of financial leverage, and when we discussed financial leverage, operating leverage was assumed to be given. Actually, the two types of leverage are interrelated. For example, if a firm *reduced* its operating leverage, this would probably lead to an *increase* in its optimal use of financial leverage. On the other hand, if it decided to *increase* its operating leverage, its optimal capital structure would probably call for *less* debt.

The theory of finance has not been developed to the point where we can actually specify simultaneously the optimal levels of operating and financial leverage. However, we can see how operating and financial leverage interact through an analysis of the *degree of leverage concept*.

DEGREE OF OPERATING LEVERAGE (DOL)

The *degree of operating leverage (DOL)*, is defined as the percentage change in operating income (or EBIT) that results from a given percentage change in sales:

$$\text{DOL} = \frac{\text{Percentage change in EBIT}}{\text{Percentage change in sales}} = \frac{\frac{\Delta \text{EBIT}}{\text{EBIT}}}{\frac{\Delta Q}{Q}} \quad (13A-1)$$

In effect, the DOL is an index number which measures the effect of a change in sales on operating income, or EBIT.

DOL can also be calculated by using Equation 13A-2, which is derived from Equation 13A-1:

$$\begin{aligned} \text{DOL}_Q &= \text{Degree of operating leverage at Point } Q \\ &= \frac{Q(P - V)}{Q(P - V) - F} \end{aligned} \quad (13A-2)$$

or, based on dollar sales rather than units,

$$\text{DOL}_S = \frac{S - VC}{S - VC - F} \quad (13A-2a)$$

Here Q is the initial units of output, P is the average sales price per unit of output, V is the variable cost per unit, F is fixed operating costs, S is initial sales in dollars, and VC is total variable costs. Equation 13A-2 is normally used to analyze a single product, such as IBM's PC, whereas Equation 13A-2a is used to evaluate an entire firm with many types of products, where "quantity in units" and "sales price" are not meaningful.

Equation 13A-2 is developed from Equation 13A-1 as follows. The change in units of output is defined as ΔQ . In equation form, $EBIT = Q(P - V) - F$, where Q is units sold, P is the price per unit, V is the variable cost per unit, and F is the total fixed costs. Since both price and fixed costs are constant, the change in EBIT is $\Delta EBIT = \Delta Q(P - V)$. The initial EBIT is $Q(P - V) - F$, so the percentage change in EBIT is

$$\% \Delta EBIT = \frac{\Delta Q(P - V)}{Q(P - V) - F}$$

The percentage change in output is $\Delta Q/Q$, so the ratio of the percentage change in EBIT to the percentage change in output is

$$DOL = \frac{\frac{\Delta Q(P - V)}{Q(P - V) - F}}{\frac{\Delta Q}{Q}} = \left(\frac{\Delta Q(P - V)}{Q(P - V) - F} \right) \left(\frac{Q}{\Delta Q} \right) = \frac{Q(P - V)}{Q(P - V) - F} \quad (13A-2)$$

Applying Equation 13A-2a to data for an illustrative firm, Hastings Inc., at a sales level of \$200,000 as shown in Table 13A-1, we find its degree of operating leverage to be 2.0:

$$\begin{aligned} DOL_{\$200,000} &= \frac{\$200,000 - \$120,000}{\$200,000 - \$120,000 - \$40,000} \\ &= \frac{\$80,000}{\$40,000} = 2.0. \end{aligned}$$

Thus, an X percent increase in sales will produce a $2X$ percent increase in EBIT. For example, a 50 percent increase in sales, starting from sales of \$200,000, will result in a $2(50\%) = 100\%$ increase in EBIT. This situation is confirmed by examining Section I of Table 13A-1, where we see that a 50 percent increase in sales, from \$200,000 to \$300,000, causes EBIT to double. Note, however, that if sales decrease by 50 percent, then EBIT will decrease by 100 percent; this is again confirmed by Table 13A-1, as EBIT decreases to \$0 if sales decrease to \$100,000.

Note also that the DOL is specific to the initial sales level; thus, if we evaluated DOL from a sales base of \$300,000, it would be different from the DOL at \$200,000 of sales:

$$\begin{aligned} DOL_{\$300,000} &= \frac{\$300,000 - \$180,000}{\$300,000 - \$180,000 - \$40,000} \\ &= \frac{\$120,000}{\$80,000} = 1.5. \end{aligned}$$

In general, if a firm is operating at close to its breakeven point, the degree of operating leverage will be high, but DOL declines the higher the base level of sales is above breakeven sales. Looking back at the top section of Table 13A-1, we see that the company's breakeven point (before consideration of financial leverage) is at sales of \$100,000. At that level, DOL is infinite:

$$\begin{aligned} DOL_{\$100,000} &= \frac{\$100,000 - \$60,000}{\$100,000 - \$60,000 - \$40,000} \\ &= \frac{\$40,000}{0} = \text{undefined but } \approx \text{infinity.} \end{aligned}$$

When evaluated at higher sales levels, DOL progressively declines.

TABLE 13A-1

**Hastings Inc.: EPS with Different Amounts of Financial Leverage
(Thousands of Dollars, except Per-Share Figures)**
I. CALCULATION OF EBIT, TOTAL ASSETS = \$200,000

Probability of indicated sales	0.2	0.6	0.2
Sales	\$100.0	\$200.0	\$300.0
Fixed costs	40.0	40.0	40.0
Variable costs (60% of sales)	60.0	120.0	180.0
Total costs (except interest)	\$100.0	\$160.0	\$220.0
Earnings before interest and taxes (EBIT)	\$ 0.0	\$ 40.0	\$ 80.0

II. SITUATION IF DEBT/ASSETS (D/A) = 0%

EBIT (from Section I)	\$ 0.0	\$ 40.0	\$ 80.0
Less interest	0.0	0.0	0.0
Earnings before taxes (EBT)	\$ 0.0	\$ 40.0	\$ 80.0
Taxes (40%)	0.0	(16.0)	(32.0)
Net income	\$ 0.0	\$ 24.0	\$ 48.0
Earnings per share (EPS) on 10,000 shares ^a	\$ 0.0	\$ 2.40	\$ 4.80
Expected EPS		\$ 2.40	
Standard deviation of EPS		\$ 1.52	
Coefficient of variation		0.63	

III. SITUATION IF DEBT/ASSETS (D/A) = 50%

EBIT (from Section I)	\$ 0.0	\$ 40.0	\$ 80.0
Less interest (0.12 × \$100,000)	12.0	12.0	12.0
Earnings before taxes (EBT)	(\$ 12.0)	\$ 28.0	\$ 68.0
Taxes (40%; tax credit on losses)	4.8	(11.2)	(27.2)
Net income	(\$ 7.2)	\$ 16.8	\$ 40.8
Earnings per share (EPS) on 5,000 shares ^a	(\$ 1.44)	\$ 3.36	\$ 8.16
Expected EPS		\$ 3.36	
Standard deviation of EPS		\$ 3.04	
Coefficient of variation		0.90	

^a The EPS figures can also be obtained using the following formula, in which the numerator amounts to an income statement at a given sales level laid out horizontally:

$$\text{EPS} = \frac{(\text{Sales} - \text{Fixed costs} - \text{Variable costs} - \text{Interest})(1 - \text{Tax rate})}{\text{Shares outstanding}} = \frac{(\text{EBIT} - I)(1 - T)}{\text{Shares outstanding}}$$

For example, with zero debt and Sales = \$200,000, EPS is \$2.40:

$$\text{EPS}_{D/A=0} = \frac{(\$200,000 - \$40,000 - \$120,000 - 0)(0.6)}{10,000} = \$2.40.$$

With 50 percent debt and Sales = \$200,000, EPS is \$3.36:

$$\text{EPS}_{D/A=0.5} = \frac{(\$200,000 - \$40,000 - \$120,000 - \$12,000)(0.6)}{5,000} = \$3.36.$$

The sales level at which EPS will be equal under the two financing policies, or the indifference level of sales, S_I , can be found by setting $\text{EPS}_{D/A=0}$ equal to $\text{EPS}_{D/A=0.5}$ and solving for S_I :

$$\text{EPS}_{D/A=0} = \frac{(S_I - \$40,000 - 0.6S_I - 0)(0.6)}{10,000} = \frac{(S_I - \$40,000 - 0.6S_I - \$12,000)(0.6)}{5,000} = \text{EPS}_{D/A=0.5}.$$

$$S = \$160,000.$$

By substituting this value of sales into either equation, we can find EPS_I , the earnings per share at this indifference point. In our example, $\text{EPS}_I = \$1.44$.

DEGREE OF FINANCIAL LEVERAGE (DFL)

Operating leverage affects earnings *before* interest and taxes (EBIT), whereas financial leverage affects earnings *after* interest and taxes, or the earnings available to common stockholders. In terms of Table 13A-1, operating leverage affects the top section, whereas financial leverage affects the lower sections. Thus, if Hastings decided to use more operating leverage, its fixed costs would be higher than \$40,000, its variable cost ratio would be lower than 60 percent of sales, and its EBIT would be more sensitive to changes in sales. *Financial leverage takes over where operating leverage leaves off, further magnifying the effects on earnings per share of changes in the level of sales.* For this reason, operating leverage is sometimes referred to as *first-stage leverage* and financial leverage as *second-stage leverage*.

The *degree of financial leverage (DFL)* is defined as the percentage change in earnings per share that results from a given percentage change in earnings before interest and taxes (EBIT), and it is calculated as follows:

$$\begin{aligned} \text{DFL} &= \frac{\text{Percentage change in EPS}}{\text{Percentage change in EBIT}} \\ &= \frac{\text{EBIT}}{\text{EBIT} - \text{I}} \end{aligned} \quad (13A-3)$$

Equation 13A-3 is developed as follows:

1. Recall that $\text{EBIT} = Q(P - V) - F$.
2. Earnings per share are found as $\text{EPS} = [(\text{EBIT} - \text{I})(1 - T)]/N$, where I is interest paid, T is the corporate tax rate, and N is the number of shares outstanding.
3. I is constant, so $\Delta I = 0$; hence, ΔEPS , the change in EPS, is

$$\Delta \text{EPS} = \frac{(\Delta \text{EBIT} - \Delta \text{I})(1 - T)}{N} = \frac{\Delta \text{EBIT}(1 - T)}{N}$$

4. The percentage change in EPS is the change in EPS divided by the original EPS:

$$\frac{\frac{\Delta \text{EBIT}(1 - T)}{N}}{\frac{(\text{EBIT} - \text{I})(1 - T)}{N}} = \left[\frac{\Delta \text{EBIT}(1 - T)}{N} \right] \left[\frac{N}{(\text{EBIT} - \text{I})(1 - T)} \right] = \frac{\Delta \text{EBIT}}{\text{EBIT} - \text{I}}$$

5. The degree of financial leverage is the percentage change in EPS over the percentage change in EBIT:

$$\text{DFL} = \frac{\frac{\Delta \text{EBIT}}{\text{EBIT} - \text{I}}}{\frac{\Delta \text{EBIT}}{\text{EBIT}}} = \left(\frac{\Delta \text{EBIT}}{\text{EBIT} - \text{I}} \right) \left(\frac{\text{EBIT}}{\Delta \text{EBIT}} \right) = \frac{\text{EBIT}}{\text{EBIT} - \text{I}} \quad (13A-3)$$

6. This equation must be modified if the firm has preferred stock outstanding.

Applying Equation 13A-3 to data for Hastings at sales of \$200,000 and an EBIT of \$40,000, the degree of financial leverage with a 50 percent debt ratio is

$$\begin{aligned} \text{DFL}_{S=\$200,000, D=50\%} &= \frac{\$40,000}{\$40,000 - \$12,000} \\ &= 1.43. \end{aligned}$$

Therefore, a 100 percent increase in EBIT would result in a $1.43(100\%) = 143$ percent increase in earnings per share. This may be confirmed by referring to the lower section of Table 13A-1, where we see that a 100 percent increase in EBIT, from \$40,000 to \$80,000, produces a 143 percent increase in EPS:

$$\% \Delta \text{EPS} = \frac{\Delta \text{EPS}}{\text{EPS}_0} = \frac{\$8.16 - \$3.36}{\$3.36} = \frac{\$4.80}{\$3.36} = 1.43 = 143\%.$$

If no debt were used, the degree of financial leverage would by definition be 1.0, so a 100 percent increase in EBIT would produce exactly a 100 percent increase in EPS. This can be confirmed from the data in Section II of Table 13A-1.

COMBINING OPERATING AND FINANCIAL LEVERAGE (DTL)

Thus far, we have seen:

1. That the greater the use of fixed operating costs as measured by the degree of operating leverage, the more sensitive EBIT will be to changes in sales, and
2. That the greater the use of debt as measured by the degree of financial leverage, the more sensitive EPS will be to changes in EBIT.

Therefore, if a firm uses a considerable amount of both operating and financial leverage, then even small changes in sales will lead to wide fluctuations in EPS.

Equation 13A-2 for the degree of operating leverage can be combined with Equation 13A-3 for the degree of financial leverage to produce the equation for the *degree of total leverage (DTL)*, which shows how a given change in sales will affect earnings per share. Here are three equivalent equations for DTL:

$$\text{DTL} = (\text{DOL})(\text{DFL}). \quad (13A-4)$$

$$\text{DTL} = \frac{Q(P - V)}{Q(P - V) - F - I}. \quad (13A-4a)$$

$$\text{DTL} = \frac{S - VC}{S - VC - F - I}. \quad (13A-4b)$$

Equation 13A-4 is simply a definition, while Equations 13A-4a and 13A-4b are developed as follows:

1. Recognize that $\text{EBIT} = Q(P - V) - F$, and then rewrite Equation 13A-3 as follows:

$$\text{DFL} = \frac{\text{EBIT}}{\text{EBIT} - I} = \frac{Q(P - V) - F}{Q(P - V) - F - I} = \frac{S - VC - F}{S - VC - F - I}. \quad (13A-3a)$$

2. The degree of total leverage is equal to the degree of operating leverage times the degree of financial leverage, or Equation 13A-2 times Equation 13A-3a:

$$DTL = (DOL)(DFL) \quad (13A-4)$$

$$= (\text{Equation 13A-2})(\text{Equation 13A-3a})$$

$$= \left[\frac{Q(P - V)}{Q(P - V) - F} \right] \left[\frac{Q(P - V) - F}{Q(P - V) - F - I} \right]$$

$$= \frac{Q(P - V)}{Q(P - V) - F - I} \quad (13A-4a)$$

$$= \frac{S - VC}{S - VC - F - I} \quad (13A-4b)$$

Applying Equation 13A-4b to data for Hastings at sales of \$200,000, we can substitute data from Table 13A-1 into Equation 13A-4b to find the degree of total leverage if the debt ratio is 50 percent:

$$\begin{aligned} DTL_{\$200,000, 50\%} &= \frac{\$200,000 - \$120,000}{\$200,000 - \$120,000 - \$40,000 - \$12,000} \\ &= \frac{\$80,000}{\$28,000} = 2.86. \end{aligned}$$

Equivalently, using Equation 13A-4, we get the same result:

$$DTL_{\$200,000, 50\%} = (2.00)(1.43) = 2.86.$$

We can use the degree of total leverage (DTL) number to find the new earnings per share (EPS_1) for any given percentage increase in sales ($\% \Delta \text{Sales}$), proceeding as follows:

$$\begin{aligned} EPS_1 &= EPS_0 + EPS_0(DTL)(\% \Delta \text{Sales}) \\ &= EPS_0[1.0 + (DTL)(\% \Delta \text{Sales})]. \end{aligned} \quad (13A-5)$$

For example, a 50 percent (or 0.5) increase in sales, from \$200,000 to \$300,000, would cause EPS_0 (\$3.36 as shown in Section III of Table 13A-1) to increase to \$8.16:

$$\begin{aligned} EPS_1 &= \$3.36[1.0 + (2.86)(0.5)] \\ &= \$3.36(2.43) \\ &= \$8.16. \end{aligned}$$

This figure agrees with the one for EPS shown in Table 13A-1.

The degree of leverage concept is useful primarily for the insights it provides regarding the joint effect of operating and financial leverage on earnings per share. The concept can be used to show the management of a business, for example, that a decision to automate a plant and to finance the new equipment with debt would result in a situation wherein a 10 percent decline in sales would produce a 50 percent decline in earnings, whereas with a different operating and financial leverage package, a 10 percent sales decline would cause earnings to decline by only 20 percent.

Having the alternatives stated in this manner gives decision makers a better idea of the ramifications of alternative actions.¹

PROBLEMS

- 13A-1** Grant Grocers has sales of \$1,000,000. The company's fixed costs total \$250,000, and its variable costs are 60 percent of sales. What is the company's degree of operating leverage? If sales increased 20 percent, what would be the percentage increase in EBIT?
- 13A-2** Arthur Johnson Inc.'s operating income is \$500,000, the company's interest expense is \$200,000, and its tax rate is 40 percent. What is the company's degree of financial leverage? If the company were able to double its operating income, what would be the percentage increase in net income?
- 13A-3** A company currently has \$2 million in sales. Its variable costs equal 70 percent of its sales, its fixed costs are \$100,000, and its annual interest expense is \$50,000.
- What is the company's degree of operating leverage?
 - If this company's operating income (EBIT) rises by 10 percent, how much will its net income increase?
 - If the company's sales increase 10 percent, how much will the company's net income increase?
- 13A-4** The Whitman Corporation will begin operations next year to produce a single product at a price of \$12 per unit. Whitman has a choice of two methods of production: Method A, with variable costs of \$6.75 per unit and fixed operating costs of \$675,000; and Method B, with variable costs of \$8.25 per unit and fixed operating costs of \$401,250. To support operations under either production method, the firm requires \$2,250,000 in assets, and it has established a debt ratio of 40 percent. The cost of debt is $k_d = 10$ percent. The tax rate is irrelevant for the problem, and fixed *operating* costs do not include interest.
- The sales forecast for the coming year is 200,000 units. Under which method would EBIT be more adversely affected if sales did not reach the expected levels? (Hint: Compare DOLs under the two production methods.)
 - Given the firm's present debt, which method would produce the greater percentage increase in earnings per share for a given increase in EBIT? (Hint: Compare DFLs under the two methods.)
 - Calculate DTL under each method, and then evaluate the firm's risk under each method.
 - Is there some debt ratio under Method A that would produce the same DTL_A as the DTL_B that you calculated in part c? (Hint: Let $DTL_A = DTL_B = 2.90$ as calculated in part c, solve for I , and then determine the amount of debt that is consistent with this level of I . Conceivably, debt could be *negative*, which implies holding liquid assets rather than borrowing.)

¹ The degree of leverage concept is also useful for investors. If firms in an industry are ranked by degree of total leverage, an investor who is optimistic about prospects for the industry might favor those firms with high leverage, and vice versa if industry sales are expected to decline. However, it is very difficult to separate fixed from variable costs. Accounting statements simply do not make this breakdown, so an analyst must make the separation in a judgmental manner. Note that costs are really fixed, variable, and "semi-variable," for if times get tough enough, firms will sell off depreciable assets and thus reduce depreciation charges (a fixed cost), lay off "permanent" employees, reduce salaries of the remaining personnel, and so on. For this reason, the degree of leverage concept is generally more useful for thinking about the general nature of the relationship than for developing precise numbers, and any numbers developed should be thought of as approximations rather than as exact specifications.