



APPENDIX

11D

COMPARING PROJECTS WITH UNEQUAL LIVES

Chapters 10 and 11 covered the basic principles of capital budgeting. In this appendix, we consider mutually exclusive projects with unequal lives. Up until now, we have treated projects as “one-shot” occurrences. However, some projects can be repeated over time. Therefore, when choosing between mutually exclusive projects, we must first determine if they can be repeated, and if so, we must take this into account when we estimate the projects’ profitability.

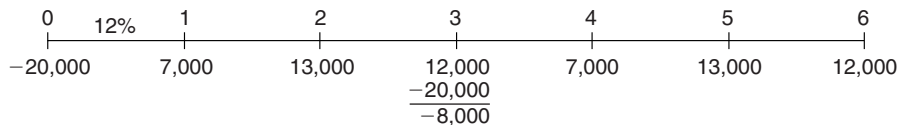
If a company is choosing between two mutually exclusive alternatives with significantly different lives, an adjustment may be necessary. For example, suppose BQC is planning to modernize its distribution center, and it is considering either a conveyor system (Project C) or some forklift trucks (Project F) for moving goods. Figure 11D-1 shows both the expected net cash flows and the NPVs for these two mutually exclusive alternatives. We see that Project C, when discounted at a 12 percent cost of capital, has the higher NPV and thus appears to be the better project.

Although the analysis in Figure 11D-1 suggests that Project C should be selected, this analysis is incomplete, and the decision to choose Project C is actually incorrect. If we choose Project F, we will have an opportunity to make a similar investment in three years, and if cost and revenue conditions remain at the Figure 11D-1 levels, this second investment will also be profitable. However, if we choose Project C, we will not have this second investment opportunity. Therefore, to make a proper comparison of Projects C and F, we could apply the **replacement chain (common life) approach**. This involves finding the NPV of Project F over a six-year period, and then comparing this extended NPV with the NPV of Project C over the same six years.

The NPV for Project C as calculated in Figure 11D-1 is already over the six-year common life. For Project F, however, we must add in a second project to extend the overall life of the combined projects to six years. Here we assume (1) that Project F’s costs and annual cash inflows will not change if the project is repeated in three years and (2) that BQC’s cost of capital will remain at 12 percent:

Replacement Chain (Common Life) Approach

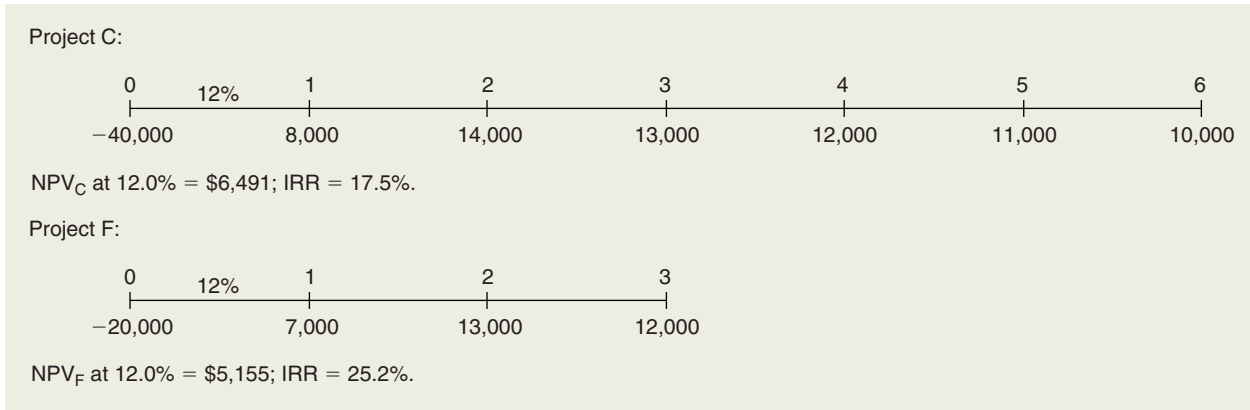
A method of comparing projects of unequal lives that assumes that each project can be repeated as many times as necessary to reach a common life span; the NPVs over this life span are then compared, and the project with the higher common life NPV is chosen.



NPV @ 12% = \$8,824; IRR = 25.2%.

The NPV of this extended Project F is \$8,824, and its IRR is 25.2 percent. (The IRR of two Project Fs is the same as the IRR for one Project F.) Since F’s \$8,824 extended NPV is greater than the \$6,491 NPV of Project C, Project F should be selected.

FIGURE 11D-1 Expected Net Cash Flows for Projects C and F

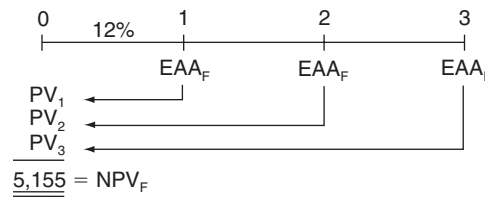


EQUIVALENT ANNUAL ANNUITY (EAA) APPROACH

Although the preceding example illustrates why an extended analysis is necessary if we are comparing mutually exclusive projects with different lives, the arithmetic is generally more complex in practice. For example, one project might have a six-year life versus a ten-year life for the other. This would require a replacement chain analysis over 30 years, the lowest common denominator of the two lives. In such a situation, it is often simpler to use a second procedure, the **equivalent annual annuity (EAA) method**, which involves three steps:

Equivalent Annual Annuity (EAA) Method
 A method which calculates the annual payments a project would provide if it were an annuity. When comparing projects of unequal lives, the one with the higher equivalent annual annuity should be chosen.

1. Find each project's NPV over its initial life. In Figure 11D-1, we found NPV_C = \$6,491 and NPV_F = \$5,155.
2. There is some constant annuity cash flow (the equivalent annual annuity [EAA]) that has the same present value as a project's calculated NPV. For Project F, here is the time line:



To find the value of EAA_F with a financial calculator, enter N = 3, k = I = 12, PV = -5155, and FV = 0, and solve for PMT. The answer is \$2,146. This level cash flow stream, when discounted back three years at 12 percent, has a present value equal to Project F's original NPV, \$5,155. The \$2,146 is called the project's "equivalent annual annuity (EAA)." The EAA for Project C can be found similarly, and it is \$1,579. Thus, Project C has an NPV which is equiv-

alent to an annuity of \$1,579 per year, while Project F's NPV is equivalent to an annuity of \$2,146.¹

3. The project with the higher EAA will always have the higher NPV when extended out to any common life. Therefore, since F's EAA is larger than C's, we would choose Project F.

The EAA method is often easier to apply than the replacement chain method, but the replacement chain method is easier to explain to decision makers. Still, the two methods lead to the same decision if consistent assumptions are used.

When should we worry about unequal life analysis? As a general rule, the unequal life issue (1) does not arise for independent projects but (2) can arise if mutually exclusive projects with significantly different lives are being compared. However, even for mutually exclusive projects, it is not always appropriate to extend the analysis to a common life. This should only be done if there is a high probability that the projects will actually be repeated at the end of their initial lives.

We should note several potentially serious weaknesses inherent in this type of analysis: (1) If inflation is expected, then replacement equipment will have a higher price. Moreover, both sales prices and operating costs will probably change. Thus, the static conditions built into the analysis would be invalid. (2) Replacements that occur down the road will probably employ new technology, which in turn might change the cash flows. This factor can be built into the replacement chain analysis but not the EAA approach. (3) It is difficult enough to estimate the lives of most projects, so estimating the lives of a series of projects is often just a speculation.

In view of these problems, no experienced financial analyst would be too concerned about comparing mutually exclusive projects with lives of, say, eight years and ten years. Given all the uncertainties in the estimation process, such projects would, for all practical purposes, be assumed to have the same life. Still, it is important to recognize that a problem exists if mutually exclusive projects have substantially different lives. When we encounter such problems in practice, we use a computer spreadsheet and build expected inflation and/or possible efficiency gains directly into the cash flow estimates, and then use the replacement chain approach (but not the equivalent annual annuity method).

PROBLEMS

- 11D-1**
Unequal lives
Haley's Graphic Designs Inc. is considering two alternative, mutually exclusive projects. Both projects require an initial investment of \$10,000 and are typical, average-risk projects for the firm. Project A has an expected life of 2 years with after-tax cash inflows of \$6,000 and \$8,000 at the end of Years 1 and 2, respectively. Project B has an expected life of 4 years with after-tax cash inflows of \$4,000 at the end of each of the next 4 years. The firm's cost of capital is 10 percent.
- a. If the projects cannot be repeated, which project will be selected, and what is its net present value?
 - b. If the projects can be repeated and there are no anticipated changes in the cash flows, which project will be selected, and what is its net present value?
- 11D-2**
Unequal lives
Cotner Clothes Inc. is considering the replacement of its old, fully depreciated knitting machine. Two new models are available: (a) Machine 190-3, which has a cost of \$190,000, a

¹ Some financial calculators have the EAA feature programmed in. For example, the HP-17B has the function in its cash flow register. One simply keys in the cash flows, enters the interest rate, and then presses the "NUS" key to get the EAA. Hewlett-Packard uses the term NUS (for "net uniform series") in lieu of the term EAA.

3-year expected life, and after-tax cash flows (labor savings and depreciation) of \$87,000 per year; and (b) Machine 360-6, which has a cost of \$360,000, a 6-year life, and after-tax cash flows of \$98,300 per year. Assume both projects can be repeated. Knitting machine prices are not expected to rise, because inflation will be offset by cheaper components (microprocessors) used in the machines. Assume that Cotner's cost of capital is 14 percent. Should the firm replace its old knitting machine, and, if so, which new machine should it use?

11D-3
Unequal lives Zappe Airlines is considering two alternative planes. Plane A has an expected life of 5 years, will cost \$100 million, and will produce net cash flows of \$30 million per year. Plane B has a life of 10 years, will cost \$132 million, and will produce net cash flows of \$25 million per year. Zappe plans to serve the route for 10 years. Inflation in operating costs, airplane costs, and fares is expected to be zero, and the company's cost of capital is 12 percent. By how much would the value of the company increase if it accepted the better project (plane)?

11D-4
Unequal lives The Fernandez Company has the opportunity to invest in one of two mutually exclusive machines that will produce a product it will need for the foreseeable future. Machine A costs \$10 million but realizes after-tax inflows of \$4 million per year for 4 years. After 4 years, the machine must be replaced. Machine B costs \$15 million and realizes after-tax inflows of \$3.5 million per year for 8 years, after which it must be replaced. Assume that machine prices are not expected to rise because inflation will be offset by cheaper components used in the machines. If the cost of capital is 10 percent, which machine should the company use?