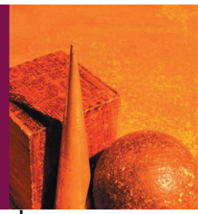

APPENDIX 6A

USING INTEREST FACTOR TABLES

In Chapter 6 we used a financial calculator to solve time value of money problems. In this Web Appendix, we discuss how we can use the interest factor tables, which are given at the back of the text in Appendix A, to solve time value of money problems. We should note that 20 years or so ago, before financial calculators and spreadsheets were widely available, the tables were used to solve most time value problems. Today, though, tables are rarely used in actual practice. Still, working through the tables can provide useful insights into various time value issues.

Solving for Future Value with Interest Tables

Future Value Interest Factor for i and n ($FVIF_{i,n}$)
The future value of $1 left on deposit for n periods at a rate of i percent per period.

The **Future Value Interest Factor for i and n ($FVIF_{i,n}$)** is defined as $(1 + i)^n$, and these factors can be found by using a regular calculator as discussed in Chapter 6 and then put into tables. Table 6A-1 is illustrative, while Table A-3 in Appendix A at the back of the book contains $FVIF_{i,n}$ values for a wide range of i and n values.

Since $(1 + i)^n = FVIF_{i,n}$, Equation 6-1, shown earlier in the text, can be rewritten as follows:

$$FV_n = PV(FVIF_{i,n}).$$

To illustrate, the FVIF for our five-year, 5 percent interest problem (discussed earlier in this chapter) can be found in Table 6A-1 by looking down the first column to

TABLE 6A-1 Future Value Interest Factors: $FVIF_{i,n} = (1 + i)^n$

PERIOD (n)	0%	5%	10%	15%
1	1.0000	1.0500	1.1000	1.1500
2	1.0000	1.1025	1.2100	1.3225
3	1.0000	1.1576	1.3310	1.5209
4	1.0000	1.2155	1.4641	1.7490
5	1.0000	**1.2763**	1.6105	2.0114
6	1.0000	1.3401	1.7716	2.3131
7	1.0000	1.4071	1.9487	2.6600
8	1.0000	1.4775	2.1436	3.0590
9	1.0000	1.5513	2.3579	3.5179
10	1.0000	1.6289	2.5937	4.0456

Period 5, and then looking across that row to the 5 percent column, where we see that $FVIF_{5\%,5} = 1.2763$. Then, the value of $100 after five years is found as follows:

$$\begin{aligned} FV_n &= PV(FVIF_{i,n}) \\ &= \$100(1.2763) = \$127.63. \end{aligned}$$

Before financial calculators became readily available (in the 1980s), such tables were used extensively, but they are rarely used today in the real world.

Solving for Present Value with Interest Tables

Present Value Interest Factor for i and n ($PVIF_{i,n}$)
The present value of $1 due n periods in the future discounted at i percent per period.

The term in parentheses in Equation 6-2, shown earlier in the text, is called the **Present Value Interest Factor for i and n,** or **$PVIF_{i,n}$,** and Table A-1 in Appendix A contains present value interest factors for selected values of i and n. The value of $PVIF_{i,n}$ for i = 5% and n = 5 is 0.7835, so the present value of $127.63 to be received after five years when the appropriate interest rate is 5 percent is $100:

$$PV = \$127.63(PVIF_{5\%,5}) = \$127.63(0.7835) = \$100.$$

Finding the Interest Rate with Interest Tables

To solve for the interest rate when n, FV, and PV are known, simply write out Equation 6-1 and susbtitute the known value into the equation as follows:

$$\begin{aligned} FV_n &= PV(1 + i)^n = PV(FVIF_{i,n}) \\ \$100 &= \$78.35(FVIF_{i,5}) \\ FVIF_{i,5} &= \$100/\$78.35 = 1.2763. \end{aligned}$$

Find the value of the FVIF as shown above, and then look across the Period 5 row in Table A-3 until you find FVIF = 1.2763. This value is in the 5% column, so the interest rate at which $78.35 grows to $100 over five years is 5 percent. (Note that Equation 6-2 will work also. However, if Equation 6-2 is used, you would solve for PVIF rather than FVIF.) This procedure can be used only if the interest rate is in the table; therefore, it will not work for fractional interest rates or where n is not a whole number. Approximation procedures can be used, but they are laborious and inexact.

Finding the Number of Periods with Interest Tables

To solve for the number of periods when i, FV, and PV are known, simply write out Equation 6-1 and substitute the known values into the equation as follows:

$$\begin{aligned} FV_n &= PV(1 + i)^n = PV(FVIF_{i,n}) \\ \$100 &= \$78.35(FVIF_{5\%,n}) \\ FVIF_{5\%,n} &= \$100/\$78.35 = 1.2763. \end{aligned}$$

Now look down the 5% column in Table A-3 until you find FVIF = 1.2763. This value is in Row 5, which indicates that it takes five years for $78.35 to grow to $100 at a 5 percent interest rate.

Solving for the Future Value of an Annuity with Interest Tables

The summation term in Equation 6-3, shown earlier in the text, is called the Future Value Interest Factor for an Annuity ($FVIFA_{i,n}$):[1]

$$FVIFA_{i,n} = \sum_{t=1}^{n} (1 + i)^{n-t}.$$

FVIFAs have been calculated for various combinations of i and n, and Table A-4 in Appendix A contains a set of FVIFA factors. To find the answer to the three-year, $100 annuity problem (discussed earlier in the chapter), first refer to Table A-4 and look down the 5% column to the third period; the FVIFA is 3.1525. Thus, the future value of the $100 annuity is $315.25:

$$FVA_n = PMT(FVIFA_{i,n})$$
$$FVA_3 = \$100(FVIFA_{5\%,3}) = \$100(3.1525) = \$315.25.$$

Solving for the Future Value of an Annuity Due with Interest Tables

In an annuity due, each payment is compounded for one additional period, so the future value of the entire annuity is equal to the future value of an ordinary annuity compounded for one additional period. Here is the solution for the annuity discussed above, assuming that the annuity payments occur at the beginning of the year:

$$FVA_n \text{ (Annuity due)} = PMT(FVIFA_{i,n})(1 + i)$$
$$= \$100(3.1525)(1.05) = \$331.01.$$

Solving for the Present Value of an Annuity with Interest Tables

Present Value Interest Factor for an Annuity ($PVIFA_{i,n}$)
The present value interest factor for an annuity of n periods discounted at i percent.

The summation term in Equation 6-4, shown earlier in the text, is called the **Present Value Interest Factor for an Annuity ($PVIFA_{i,n}$),** and values for the term at different values of i and n are shown in Table A-2 at the back of the book. Here is the equation:

$$PVA_n = PMT(PVIFA_{i,n}).$$

[1] Another form for this equation is as follows:

$$FVIFA_{i,n} = \frac{(1 + i)^n - 1}{i}.$$

This form is found by applying the algebra of geometric progressions. This equation is useful in situations when the required values of i and n are not in the tables and no financial calculator or computer is available.

To find the answer to the three-year, \$100 annuity problem (discussed earlier in the chapter), simply refer to Table A-2 and look down the 5% column to the third period. The PVIFA is 2.7232, so the present value of the \$100 annuity is \$272.32:

$$PVA_n = PMT(PVIFA_{i,n})$$
$$PVA_3 = \$100(PVIFA_{5\%,3}) = \$100(2.7232) = \$272.32.$$

Solving for the Present Value of an Annuity Due with Interest Tables

In an annuity due, each payment is discounted for one less period. Since its payments come in faster, an annuity due is more valuable than an ordinary annuity. This higher value is found by multiplying the PV of an ordinary annuity by (1 + i). To find the present value of the annuity discussed above assuming that annuity payments occur at the beginning of the year, we use the following equation:

$$PVA_n \text{ (Annuity due)} = PMT(PVIFA_{i,n})(1 + i)$$
$$= \$100(2.7232)(1.05) = \$285.94.$$