



## APPENDIX

## 15A

ARITHMETIC VERSUS  
GEOMETRIC AVERAGES

The *arithmetic average* gives us an indication of the “typical” growth rate for any given year, and it is calculated as the sum of the annual growth rates divided by the number of years:

$$\text{Arithmetic average} = \Sigma(g_i)/N,$$

where

$g_i$  = annual growth rate in year  $i$ .  
 $N$  = the number of years.

The *geometric average*, which takes into account the effects of compounding, provides a better indication of the company’s long-run growth rate. The geometric average is calculated as follows:

$$\text{Geometric average} = [\pi (1 + g_i)]^{1/N} - 1,$$

where again

$g_i$  = annual growth rate in year  $i$ .  
 $N$  = the number of years.  
 $\pi$  = the product of elements.<sup>1</sup>

We can illustrate these two concepts, and the differences between them, with the following set of sales data for a hypothetical company:

YEAR	SALES	GROWTH RATE
2000	\$100.0	—
2001	130.0	30%
2002	91.0	−30
2003	118.3	30

Note that it takes four years of data to calculate three growth rates. The arithmetic average is simply  $(30\% + (-30\%) + 30\%)/3 = 10.0\%$ . The geometric average is  $[(1.3)(0.7)(1.3)]^{1/3} - 1 = 5.76\%$ . Which growth rate is “better”? If you want to use the past to predict the future, then the geometric growth rate is better. To see why, take the beginning sales of \$100, apply the two growth rates, and see which one gets us closer to the ending value of \$118.3:

**Arithmetic:** Ending value =  $\$100(1.1)(1.1)(1.1) = \$133.1$ .

**Geometric:** Ending value =  $\$100(1.0576)(1.0576)(1.0576) = \$118.3$ .

<sup>1</sup>  $\pi$  is a symbol that indicates that a series of numbers should be multiplied together. In other words,  $\pi$  is to multiplication as  $\Sigma$  is to addition.

The geometric average takes account of compounding and thus gets us from the beginning value to the actual ending value, hence it is better for predicting future results.

If the annual growth rates are not very different from year to year, then the arithmetic and geometric averages will be similar, so it does not make a lot of difference which one is used. However, if the annual rates vary widely, then it does matter, and the geometric average should be used.