APPENDIX 7A

ZERO COUPON BONDS

To understand how zeros are used and analyzed, consider the zeros that are going to be issued by Vandenberg Corporation, a shopping center developer. Vandenberg is developing a new shopping center in San Diego, California, and it needs $50 million. The company does not anticipate major cash flows from the project for about five years. However, Pieter Vandenberg, the president, plans to sell the center once it is fully developed and rented, which should take about five years. Therefore, Vandenberg wants to use a financing vehicle that will not require cash outflows for five years, and he has decided on a five-year zero coupon bond, with a maturity value of $1,000.

Vandenberg Corporation is an A-rated company, and A-rated zeros with five-year maturities yield 6 percent at this time (five-year coupon bonds also yield 6 percent). The company is in the 40 percent federal-plus-state tax bracket. Pieter Vandenberg wants to know the firm's after-tax cost of debt if it uses 6 percent, five-year maturity zeros, and he also wants to know what the bond's cash flows will be. Table 7A-1 provides an analysis of the situation, and the following numbered paragraphs explain the table itself.

TABLE 7A-1 **Analysis of a Zero Coupon Bond from Issuer's Perspective**

BASIC DATA

Maturity value	$1,000
k_d	6.00%, annual compounding
Maturity	5 years
Corporate tax rate	40.00%
Issue price	$747.26

ANALYSIS

	0 (6%)	1	2	3	4	5 Years
(1) Year-end accrued value	$747.26	$792.10	$839.62	$890.00	$943.40	$1,000.00
(2) Interest deduction		44.84	47.52	50.38	53.40	56.60
(3) Tax savings (40%)		17.94	19.01	20.15	21.36	22.64
(4) Cash flow to Vandenberg	+747.26	+17.94	+19.01	+20.15	+21.36	−977.36
After-tax cost of debt	3.60%					

Face value of bonds the company must issue to raise $50 million = Amount needed/Issue price as % of par
= $50,000,000/0.74726
≈ $66,911,000.

1. The information in the "Basic Data" section, except the issue price, was given in the preceding paragraph, and the information in the "Analysis" section was calculated using the known data. The maturity value of the bond is always set at $1,000 or some multiple thereof.
2. The issue price is the PV of $1,000, discounted back five years at the rate k_d = 6%, annual compounding. Using a financial calculator, we input N = 5, I = 6, PMT = 0, and FV = 1000, then press the PV key to find PV = $747.26. Note that $747.26, compounded annually for five years at 6 percent, will grow to $1,000 as shown by the time line on Line 1 in Table 7A-1.
3. The accrued values as shown on Line 1 in the analysis section represent the compounded value of the bond at the end of each year. The accrued value for Year 0 is the issue price; the accrued value for Year 1 is found as $747.26(1.06) = $792.10; the accrued value at the end of Year 2 is 747.26(1.06)^2$ = $839.62; and, in general, the value at the end of any Year n is

$$\text{Accrued value at the end of Year n} = \text{Issue price} \times (1 + k_d)^n. \quad \textbf{(7A-1)}$$

4. The interest deduction as shown on Line 2 represents the increase in accrued value during the year. Thus, interest in Year 1 = $792.10 − $747.26 = $44.84. In general,

$$\text{Interest in Year n} = \text{Accrued value}_n - \text{Accrued value}_{n-1}. \quad \textbf{(7A-2)}$$

   This method of calculating taxable interest is specified in the Tax Code.
5. The company can deduct interest each year, even though the payment is not made in cash. This deduction lowers the taxes that would otherwise be paid, producing the following tax savings:

$$\begin{aligned}\text{Tax savings} &= (\text{Interest deduction})(T). \quad \textbf{(7A-3)}\\ &= \$44.84(0.4)\\ &= \$17.94 \text{ in Year 1.}\end{aligned}$$

6. Line 4 represents cash flows on a time line; it shows the cash flow at the end of Years 0 through 5. At Year 0, the company receives the $747.26 issue price. The company also has positive cash inflows equal to the tax savings during Years 1 through 4. Finally, in Year 5, it must pay the $1,000 maturity value, but it gets one more year of interest tax savings. Therefore, the net cash flow in Year 5 is −$1,000 +$22.64 = −$977.36.
7. Next, we can determine the after-tax cost (or after-tax yield to maturity) of issuing the bonds. Since the cash flow stream is uneven, the after-tax yield to maturity is found by entering the after-tax cash flows, shown in Line 4 of Table 7A-1, into the cash flow register and then pressing the IRR key on the financial calculator. The IRR is the after-tax cost of zero coupon debt to the company. Conceptually, here is the situation:

$$\sum_{t=1}^{n} \frac{CF_n}{(1 + k_{d(AT)})^n} = 0. \quad \textbf{(7A-4)}$$

$$\frac{\$747.26}{(1+k_{d(AT)})^0} + \frac{\$17.94}{(1+k_{d(AT)})^1} + \frac{\$19.01}{(1+k_{d(AT)})^2} + \frac{\$20.15}{(1+k_{d(AT)})^3} + \frac{\$21.36}{(1+k_{d(AT)})^4} + \frac{-\$977.36}{(1+k_{d(AT)})^5} = 0.$$

   The value $k_{d(AT)}$ = 0.036 = 3.6%, found with a financial calculator, produces the equality, and it is the cost of this debt. (Input in the cash flow register

$CF_0 = 747.26$, $CF_1 = 17.94$, and so forth, out to $CF_5 = -977.36$. Then press the IRR key to find $k_d = 3.6\%$.)

8. Note that $k_d(1 - T) = 6\%(0.6) = 3.6\%$. As we will see in Chapter 9, the cost of capital for regular coupon debt is found using the formula $k_d(1 - T)$. Thus, there is symmetrical treatment for tax purposes for zero coupon and regular coupon debt; that is, both types of debt use the same after-tax cost formula. This was Congress's intent, and it is why the Tax Code specifies the treatment set forth in Table 7A-1.[1]

The purchaser of a zero coupon bond must calculate interest income on the bond in the same manner as the issuer calculates the interest deduction. Table 7A-2 shows the resulting tax payments for an investor in the 27 percent tax bracket who purchases the Vandenberg bond. Given this tax treatment, investors pay taxes in each year even though they don't receive any cash flows until the bond is sold or matures, a situation that many investors find unattractive. Consequently, because of the tax situation pension funds and other tax-exempt entities buy most zero coupon bonds. Individuals do, however, buy taxable zeros for their Individual Retirement Accounts (IRAs). Also, state and local governments issue "tax-exempt muni zeros" that are purchased by individuals in high tax brackets.

Not all original issue discount bonds (OIDs) have zero coupons. For example, Vandenberg might have sold an issue of five-year bonds with a 5 percent coupon at a time when other bonds with similar ratings and maturities were yielding 6 percent. Such bonds would have had a value of $957.88:

$$\text{Bond value} = \sum_{t=1}^{5} \frac{\$50}{(1.06)^t} + \frac{\$1{,}000}{(1.06)^5} = \$957.88.$$

[1] Note too that we have analyzed the bond as if the cash flows accrued annually. Generally, to facilitate comparisons with semiannual payment coupon bonds, the analysis is conducted on a semiannual basis.

TABLE 7A-2 **Analysis of a Zero Coupon Bond from Investor's Perspective**

BASIC DATA

Maturity value	$1,000
k_d	6.00%, annual compounding
Maturity	5 years
Personal tax rate	27.00%
Issue price	$747.26

ANALYSIS

	0 (6%)	1	2	3	4	5 Years
(1) Year-end accrued value	$747.26	$792.10	$839.62	$890.00	$943.40	$1,000.00
(2) Interest income		44.84	47.52	50.38	53.40	56.60
(3) Tax payment (27%)		12.11	12.83	13.60	14.42	15.28
(4) Cash flow to investor	−747.26	−12.11	−12.83	−13.60	−14.42	+984.72
After-tax return	4.38%					

If an investor had purchased these bonds at a price of $957.88, the yield to maturity would have been 6 percent. The discount of $1,000 − $957.88 = $42.12 would have been amortized over the bond's five-year life, and it would have been handled by both Vandenberg and the bondholders exactly as the discount on the zeros was handled.

Thus, zero coupon bonds are just one type of original issue discount bond. Any nonconvertible bond whose coupon rate is set below the going market rate at the time of its issue will sell at a discount, and it will be classified (for tax and other purposes) as an OID bond.

Shortly after corporations began to issue zeros, investment bankers figured out a way to create zeros from U.S. Treasury bonds, which at the time were issued only in coupon form. In 1982, Salomon Brothers bought $1 billion of 12 percent, 30-year Treasuries. Each bond had 60 coupons worth $60 each, which represented the interest payments due every six months. Salomon then in effect clipped the coupons and placed them in 60 piles; the last pile also contained the now "stripped" bond itself, which represented a promise of $1,000 in the year 2012. These 60 piles of U.S. Treasury promises were then placed with the trust department of a bank and used as collateral for "zero coupon U.S. Treasury Trust Certificates," which are, in essence, zero coupon Treasury bonds. Treasury zeros are, of course, safer than corporate zeros, so they are very popular with pension fund managers. In response to this demand, the Treasury has also created its own "Strips" program, which allows investors to purchase zeros electronically.

Corporate (and municipal) zeros are generally callable at the option of the issuer, just like coupon bonds, after some stated call protection period. The call price is set at a premium over the accrued value at the time of the call. Stripped U.S. Treasury bonds (Treasury zeros) generally are not callable because the Treasury normally sells noncallable bonds. Thus, Treasury zeros are completely protected against reinvestment risk (the risk of having to invest cash flows from a bond at a lower rate because of a decline in interest rates).

PROBLEMS

7A-1 Zero coupon bonds

A company has just issued 4-year zero coupon bonds with a maturity value of $1,000 and a yield to maturity of 9 percent. The company's tax rate is 40 percent. What is the after-tax cost of debt for the company?

7A-2 Zero coupon bonds

An investor in the 27 percent bracket purchases the bond discussed in Problem 7A-1. What is the investor's after-tax return?

7A-3 Zero coupon bonds and EAR

Assume that the city of Tampa sold tax-exempt (muni), zero coupon bonds 5 years ago. The bonds had a 25-year maturity and a maturity value of $1,000 when they were issued, and the interest rate built into the issue was a nominal 10 percent, but with semiannual compounding. The bonds are now callable at a premium of 10 percent over the accrued value. What effective annual rate of return would an investor who bought the bonds when they were issued and who still owns them earn if they are called today?