APPENDIX 6B

CONTINUOUS COMPOUNDING AND DISCOUNTING

In Chapter 6 we dealt only with situations where interest is added at discrete intervals—annually, semiannually, monthly, and so forth. In some instances, though, it is possible to have instantaneous, or *continuous*, growth. In this appendix, we discuss present value and future value calculations when the interest rate is compounded continuously.

CONTINUOUS COMPOUNDING

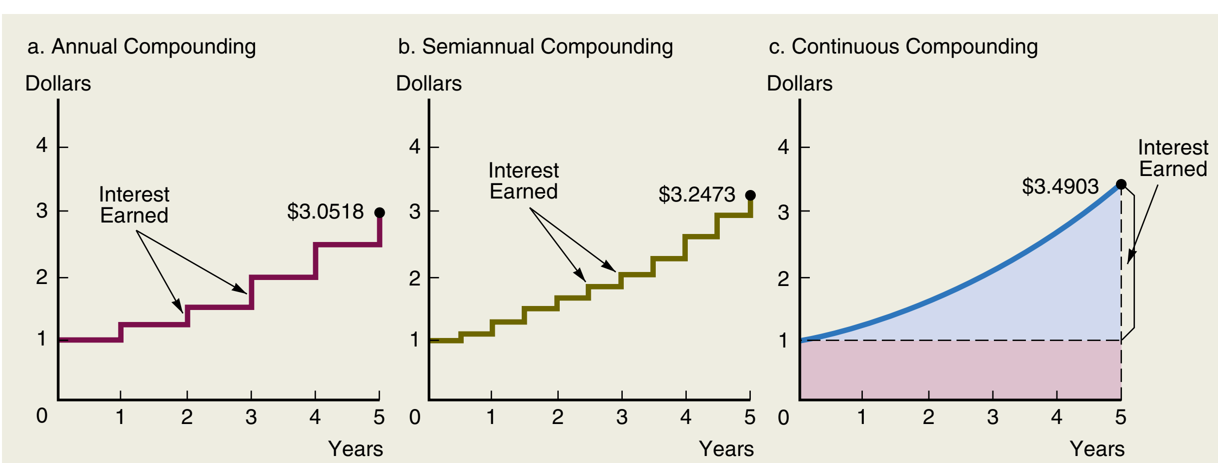
Continuous Compounding
A situation in which interest is added continuously rather than at discrete points in time.

The relationship between discrete and **continuous compounding** is illustrated in Figure 6B-1. Panel a shows the annual compounding case, where interest is added once a year; Panel b shows the situation when compounding occurs twice a year; and Panel c shows interest being earned continuously. As the graphs show, the more frequent the compounding period, the larger the final compounded amount because interest is earned on interest more often.

Equation 6-9 in the chapter can be applied to any number of compounding periods per year:

$$\text{More frequent compounding: } FV_n = PV\left(1 + \frac{i_{Nom}}{m}\right)^{mn}. \qquad \textbf{(6-9)}$$

FIGURE 6B-1 **Annual, Semiannual, and Continuous Compounding: Future Value with i = 25%**



To illustrate, let PV = \$100, i = 10%, and n = 5. At various compounding periods per year, we obtain the following future values at the end of five years:

$$\text{Annual: } FV_5 = \$100\left(1 + \frac{0.10}{1}\right)^{1(5)} = \$100(1.10)^5 = \$161.05.$$

$$\text{Semiannual: } FV_5 = \$100\left(1 + \frac{0.10}{2}\right)^{2(5)} = \$100(1.05)^{10} = \$162.89.$$

$$\text{Monthly: } FV_5 = \$100\left(1 + \frac{0.10}{12}\right)^{12(5)} = \$100(1.0083)^{60} = \$164.53.$$

$$\text{Daily: } FV_5 = \$100\left(1 + \frac{0.10}{365}\right)^{365(5)} = \$164.86.$$

We could keep going, compounding every hour, every minute, every second, and so on. At the limit, we could compound every instant, or *continuously*. The equation for continuous compounding is

$$FV_n = PV(e^{in}). \qquad \textbf{(6B-1)}$$

Here e is the value 2.7183. . . . If \$100 is invested for five years at 10 percent compounded continuously, then FV_5 is calculated as follows:[1]

$$\text{Continuous: } FV_5 = \$100[e^{0.10(5)}] = \$100(2.7183\ .\ .\ .)^{0.5} = \$164.87.$$

Continuous Discounting

Equation 6B-1 can be transformed into Equation 6B-2 and used to determine present values under continuous discounting:

$$PV = \frac{FV_n}{e^{in}} = FV_n(e^{-in}). \qquad \textbf{(6B-2)}$$

Thus, if \$1,649 is due in 10 years, and if the appropriate *continuous* discount rate, i, is 5 percent, then the present value of this future payment is

$$PV = \frac{\$1{,}649}{(2.7183\ .\ .\ .)^{0.5}} = \frac{\$1{,}649}{1.649} = \$1{,}000.$$

[1] Calculators with exponential functions can be used to evaluate Equation 6B-1. For example, with an HP-10BII you would type .5, then press the e^x key to get 1.6487, and then multiply by \$100 to get \$164.87.